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Radiated Field From a Thin Half-Wave Dipole Excited By a Single-Cycle Sinusoid

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The primary objective is to study the behavior of the radiated electric field from a half-wave dipole in free space excited by a single-cycle sinusoidal voltage. In order to accomplish this we first compute analytically the radiate electric field from a thin electric dipole antenna, excite at the center by a sinusoidal voltage of finite duration T using an approximation. This approximation consists of retaining only the zero-order solution for the current along the antenna in the frequency domain. Then the time-dependent current as well as the radiated electric field are calculated. For a matched system, the time dependent current along each half-section of the dipole is made up of a signal from the feed point and a signal reflected from the end. On the other hand, the radiated electric field consists of four signals exhibiting that radiation takes place only from the discontinuities of the dipole antenna. For a half-wave dipole at the carrier frequency f_0, the time dependent current along the antenna as well as the radiated field, due to a single-cycle sinusoidal excitation, are plotted as functions of a normalized time. Both current and the radiated electric field are extended in time due to the reflection of current from the end points of the dipole antenna. It appears that the spectral bandwidth of the radiated electric field is narrower than that of the exciting voltage pulse. This phenomenon may be due to the filtering effect of the half-wave dipole.

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CONTENTS

RADIATED FIELD FROM A THIN HALF-WAVE DIPOLE EXCITED BY A SINGLE-CYCLE SINUSOID

1. INTRODUCTION

In many impulse radars carrier-free pulses are used, since the radiated signals have broad frequency bandwidth which may be helpful in target detection. A single-cycle sinusoidal signal also has such a broadband property. Consequently, we will study the behavior of a radiated signal from an electric dipole antenna excited by a single-cycle sinusoid. The analytical approach to the solution of this problem is analogous to the transient radiation problems.

It is well known that if a transient waveform is applied to the input of a network consisting of lumped circuit parameters, R, L, and C, the signal at the output becomes distorted, i.e., it is not a perfect replica of the input waveform. The main reason behind this phenomenon is that the impedance, current and voltage at the output terminals of the network are functions of frequency; in other words, the network has a finite bandwidth. Consequently, the response of the network is different for different frequencies, which exist in a transient signal. Similarly, an antenna can also be represented, in principle, by R, L, and C for a given frequency. In this situation also one can, therefore, intuitively expect that the radiated waveform from an antenna excited by a given shape of a pulse will be distorted. Since in several radar systems pulses are used for the detection and identification of a target and since a successful identification of a target may require the knowledge of the shape of the returned pulses, it becomes important to know beforehand the shape of the incident waveform on the target, remembering the fact that this incident waveform is not the same as the pulse applied to the transmitting antenna. Once the shape of the incident pulse (which is the radiated signal from the transmitting antenna) is known, then the returned or scattered signal from the target can be determined in principle.

As mentioned earlier our primary interest lies in determining the shape of the radiated signal from a half-wave dipole antenna in free space excited by a single-cycle sinusoidal voltage applied across the antenna feed point. Although transient radiation from antennas have been studied extensively in the literature [1,2] using various shapes of the exciting pulses, it appears that the radiation from an electric dipole excited by a single-cycle sinusoidal voltage pulse has not been investigated previously. Since Refs. [1,2] discuss several methods of analysis of transients problems and provide exhaustive related references, there is no need of reviewing different methods and models here.

In general, a transient problem of this type cannot be solved rigorously by any analytical method. However, fairly accurate result can be obtained numerically [3,4], which is also time consuming. For this reason we shall use some approximations which will enable us to calculate the time dependent radiated electric field analytically. This approximation consists of the assumption of a thin antenna, where the current appears to be concentrated along the antenna axis. In addition, only the zero-order solution of the integro-differential equation for the antenna current in the frequency or Fourier-transform domain is retained using these approximations. By allowing a non-zero internal impedance (real) of the pulse generator, a modified zero-order antenna current in the frequency domain is re-calculated. Then the time dependent antenna current as well as the radiated electric field pulses are determined. When the generator impedance is matched to that of the dipole, it is found that on each half-length of the antenna there are two signal currents, one starts directly from the

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center of the feed point and the other arrives after undergoing a single reflection from the end-point of the antenna. Due to this reflection, both the total signal current and the total radiated electric field are extended in time. The electric field consists of four signals for all angles of observation, exhibiting the fact that radiation takes place only from the discontinuities (center and end points) of the antenna. Furthermore, the total current at a given point $|z|$ on the antenna shows an even symmetry about the point $f_0 t = 1{\cdot}5/2$, whereas the total radiated electric field has an odd symmetry about normalized time $f_0 t^* = 1{\cdot}5/2$, where f_0 is the frequency of the sinusoidal exciting voltage and t and t^* are real and retarded times respectively.

Numerical results for a half-wave dipole designed for f_0 show that the duration of the radiated electric field is increased by $0{\cdot}5f/_0$, when the duration of the input voltage pulse is an integer multiple of T_0. $T_0 = 1/f_0$ is the width of a single-cycle. The spectral bandwidth of the radiated electric field appears to be narrower than that of the applied voltage. The reason for this phenomenon may be due to the filtering effect of the half-wave dipole.

Although the results obtained here are based on the above mentioned approximation, it is hoped that the present results will be useful in interpreting more accurate results which one could compute by numerical method(s).

Our results are given in the next section. The details of the analysis are deferred to the appendix.

2. SOME NUMERICAL RESULTS FOR A HALF-WAVE DIPOLE

When the input voltage is a sinusoid of a single-cycle, then the pulse length T_0 is given by $\omega_0 T_0 = 2\pi$, where $\omega_0 = 2\pi f_0$ is the sinusoidal carrier frequency (angular). For a half-wave center-fed dipole of total length $2h$ designed for the carrier frequency f_0 one finds $\omega_0 h/c = \pi/2$ or $h/c = T_0/4$. Figure 2 shows the normalized time dependent modified current $I_{0,M}(z,t)/[V_0/2Z_0]$ along the half-wave dipole as a function of a normalized time tf_0, where t is the real time. The horizontal line passing through 0 shows that no current flows at the antenna end points $|z| = h$. The duration of the total current pulse observed at $|z| = 0$ is $1{\cdot}5T_0$, whereas the exciting voltage pulse width is T_0. However, as the observation point $|z|$ along the dipole increases, the duration of the current pulses decreases, approaching zero as $|z| \rightarrow h$. The current pulse at any point $|z|$ is even symmetric about the normalized time $tf_0 = 1{\cdot}5/2$, when the current attains a negative maximum for all $|z| \neq h$. This negative maximum appears to be higher in magnitude than the two positive maxima for all $|z| \neq h$. In particular, the two positive maxima at $z = 0$ occur at $tf_0 = 0{\cdot}25$ and $1{\cdot}25$ respectively.

In Fig. 3 the normalized time dependent radiated electric field, $E_\theta(r, \theta, t)/[V_0 \zeta_0/(4\pi Z_0 r)]$ is shown as a function of normalized retarded time $t^* f_0$ for four different angle of observations θ = 30°, 45°, 60° and 90°, which are measured from the axis, [see Fig. 1], of the dipole. As expected the broadside (θ = 90°) field is the strongest one. For all angles θ the electric field shows an odd symmetry about $t^* f_0 = 1{\cdot}5/2$. For each angle of observation the magnitude of the normalized electric field is maximum around the time $f_0 t^* = 0{\cdot}5$ and 1. Although the exciting sinusoidal voltage has a length T_0 (for a single-cycle), the composite radiated time-dependent electric field (like the current along the dipole at a given point) has a time duration of $1{\cdot}5/f_0$, i.e., the radiated signal is extended 1·5 times compared to the exciting voltage. This phenomenon of broadenin of the radiated field is caused by the reflection of current from the end points of the dipole. It may be noticed that if the duration of the input voltage is nT_0, an integer multiple of T_0, then [see Fig. 4] the duration of the corresponding radiated field becomes $(n + 1/2)T_0$ only. This increased duration appears as distortions in the beginning and the end of the respective radiated signals. Figure 5 shows the frequency response of the radiated electric field at broadside as a function of the normalized frequency f/f_0 for four different input signals of durations T_0, $2T_0$, $3T_0$ and $4T_0$. The maximum amplitudes of these radiated fields occur at $f > f_0$, but they approach $f/f_0 = 1$ as the duration of the input voltage is increased. This phenomenon can also be observed from Figs. 6 and 7, where the frequency responses of the input applied voltage and the radiated electric field at broadside are compared. Figures 6 and 7 drawn for $2h/a = 75$, indicate (see Eq. 6 of Appendix) that the normalized radiated electric field $|r \hat{E}_\theta(r, \theta, \omega)|$, which attenuates more for higher values of $\Omega = 2\ln(2h/a)$ has a frequency bandwidth relatively narrower than that of the applied voltage. The reason for this narrow band may be due to the thin antenna, which in general has a narrow band. It may also be noticed that the antenna is attenuating more at lower frequencies than at higher ones.

3. CONCLUSION

A modified zero-order approximate solution is used for the frequency or Fourier-transform domain current distribution along a dipole antenna. It is shown that when the dipole is matched to and is excited by a sinusoidal pulse generator, delivering a voltage $\sin \omega_0 t$ of duration T, the corresponding time dependent antenna current as well as the radiated electric field are broadened in time longer than T. This phenomenon is caused by the reflection of time dependent current from the end points of the antenna. In particular, when the duration of the input voltage is an integer multiple of T_0, where T_0 is the period of a single-cycle, the duration of the radiated electric field is increased by $0{\cdot}5T$. At a given point $|z|$ on any half-section of the dipole the total current consists of two pulses, one is incident from $|z| = 0$ and the other arrives at $|z|$ after undergoing a reflection from the end point. The duration of the current pulse along the antenna decreases as the observation point on the antenna increases. The current pulse at a given point $|z|$ of the antenna possesses an even symmetry about the dimensionless time $tf_0 = 1{\cdot}5/2$. The corresponding radiated electric field consists of four signal, each of duration $1/f_0$, which overlap in time. Of these four signals, the first one resembles very much the input sinusoidal voltage, but retarded, and is radiated directly from the feed point $z = 0$ of the dipole antenna. The second and the third signals radiate from the end points, $z = h$ and $z = -h$ respectively of the dipole, and, therefore, delay in time with respect to the first one by $h(1 - \cos\theta)/c$ and $h(1 + \cos\theta)/c$. The direction of the observation, given by θ, is measured from the axis of the half-wave dipole. The fourth signal delayed in time by $2h/c$ with respect to the first one, is also radiated from the feed point $z = 0$, after the current undergoes reflection from two ends $z = \pm h$. In otherwords, one half of the fourth signal is contributed by the current which traveled from $z = 0$ to $z = h$ and then back to $z = 0$, and the other half is due to the current traveling from $z = 0$ to $z = -h$ and then back to $z = 0$ (see Eq. 13 in appendix). This observation indicates that radiation originates from the discontinuous points of the antenna. It may be noted also that the total radiated electric field for any angle of observation has an odd symmetry about the normalized retarded time $t^*f_0 = 1{\cdot}5/2$. The spectra of the applied voltage signal and the corresponding radiated electric field do not have their maxima at the same frequency. The peak value of the frequency spectra of the radiated electric field occurs at a frequency $f > f_0$. If the duration of the applied signal is increased, the peak value of the radiated spectra approaches the peak of the applied voltage spectrum at the carrier frequency $f = f_0$. The bandwidth of the radiated normalized electric field is narrower than that of the applied voltage. The reason may be that the antenna is assumed to be thin, which in general, has a narrow bandwidth. The antenna attenuates more at lower frequencies than at higher ones.

Since the results presented here are based on an approximate expression of the current, some of the observations noted above may not be precise. Nevertheless, it is believed that they will be helpful in interpreting more accurate results which may be obtained by numerical methods.

Appendix

ANALYSIS

Let us consider a center-fed slender cylindrical antenna of total length $2h$ [Fig. 1] in free space, excited by a sinusoidal pulse generator. The coordinate system has the origin at the center of the antenna. Initially we assume that the generator has zero internal impedance and the antenna is a perfect conductor. it can be shown [5,6] that in the Fourier-transform domain (frequency domain) the current distrubution $\hat{I}(z, \omega)$ along the antenna (oriented in the z-direction) obeys the following integral equation.

$$(i\mu_0 c/4\pi) \int_{-h}^{h} \hat{I}(z,\omega) \{\exp(-i\beta R)/R\} dz' = c_1(\omega) \cos \beta z + \frac{\hat{V}(\omega)}{2} \sin (\beta |z|), \qquad (1)$$

where

$$R = [(z - z')^2 + a^2]^{1/2} \qquad (2a)$$

$$a = \text{radius of the thin cylinder}, \qquad (2b)$$

$$c = \text{velocity of light}, \qquad (2c)$$

$$\beta = \omega/c \qquad (2d)$$

Both z and z' lie along the axis of the antenna, where the current is assumed to be concentrated. The constant $c_1(\omega)$ is to be determined from the boundary conditions at the ends of the antenna. $\hat{V}(\omega)$, the Fourier-transform of the sinusoidal input voltage pulse is given by

$$\hat{V}(\omega) = V_0 \int_{-\infty}^{\infty} \sin \omega_0 t \, [U(t) - U(t - T)] \, e^{-i\omega t} dt$$

$$= V_0 \int_{0}^{T} \sin \omega_0 t \, e^{-i\omega t} dt$$

$$= -[V_0 \omega_0/(\omega^2 - \omega_0^2)] \cdot [1 - F(\omega, T) \, e^{-i\omega T}], \qquad (3a)$$

where

$$F(\omega, T) = \cos \omega_0 T + i(\omega/\omega_0) \sin \omega_0 T, \qquad (3b)$$

$$T = \text{duration of the pulse}$$

$$\omega_0 = \text{carrier frequency (angular) of the pulse}, \qquad (3b)$$

$$V_0 = \text{amplitude of the voltage which represents a slice generator at } z = 0 \qquad (3c)$$

and where $U(t)$ and $U(t - T)$ are unit step functions. Several simplifying assumptions as mentioned earlier are made [5,6] in obtaining the integral Eq. (1), but yet no closed form solution is available. Equation (1) then can be solved either analytically by an iteration method [5,6] which produces an approximate analytical expression, or numerically [5]. There is an advantage in obtaining an analytical expression for the current $\hat{I}(z,\omega)$, since the corresponding result could be interpreted more easily. The analytical expression obtained by the method of iteration contains terms with integer powers of $(1/\Omega)$, where $\Omega = 2 ln\ (2h/a)$ is much larger than unity. The degree of approximation depends on retaining the highest power of $(1/\Omega)$. The lowest order approximation containing only the first power of $(1/\Omega)$ is known as the zero-order approximation [6]. The zero-order solution of (1) has the simple form [5,6].

$$\hat{I}_0(z,\omega) = [i2\pi/(\Omega\zeta_0)] \cdot \hat{V}(\omega) \cdot [\sin\beta(h - |z|)/\cos\beta h], \tag{4}$$

where $\zeta_0 = \mu_0 c$ = free space impedance. In the first-order approximation, the factor $\sin\beta(h - |z|)/\cos\beta h$ is replaced by $[\sin\beta(h - |z|) + b_1(z,\omega)/\Omega]/[\cos\beta h + d_1(h,\omega)/\Omega]$. Explicit expressions for $b_1(z,\omega)$ and $d_1(h,\omega)$ can be found in Refs. [5,6]. Since a pulse contains a broad frequency spectrum, one should take higher-order approximations of the current $\hat{I}(z,\omega)$, so that it represents accurately the desired frequency spectra. However, in order to compute the corresponding current as well as the radiated field in time domain, one needs to perform the respective inverse Fourier-transforms, which can only be evaluated numerically. Consequently, one faces the same numerical computations problem again. Even the use of the first-order approximation for $\hat{I}(z,\omega)$, requires extensive numerical computation for obtaining the corresponding time domain result; and, therefore, a simple interpretation of the time domain result may not be possible. This is one of the main reasons why we shall consider here only the zero-order approximation given by (4), the use of which permits the analytical computations of the corresponding time domain current and radiated field exactly; and consequently simple interpretations of the results can be offered. Another justification of choosing the zero-order approximation of $\hat{I}(z,\omega)$ can be stated in the following way. The radiating dipole antenna is designed to be very near resonant at the carrier frequency ω_0 of the input signal. The antenna acts as a frequency filtering device, therefore, most of the radiated energy is expected to be concentrated around the frequency ω_0. It is also known [5] that the current in the frequency domain of a thin dipole antenna can be adequately represented by the above expression (4). It is, therefore, expected that even if one wishes to compute the time-domain current and transient radiated field more accurately by numerical means (such as the moment method), the present result obtained by using (4) could be used as a helpful guide for the purpose of comparison, since the present result will exhibit the most salient characteristics of the transient radiation of a sinusoidal pulse from a dipole antenna.

With the foregoing explanation we now proceed to use the zero-order approximation of the current given by (4) in the frequency domain. This expression resembles the current distribution along an open-circuited transmission line of length h and characteristic impedance $Z_0 = \Omega\zeta_0/2\pi$, excited by a voltage generator (with zero internal impedance) delivering a voltage which has a Fourier transform $\hat{V}(\omega)$ [5]. Since the antenna is near resonant at the frequencies given by $\cos\beta h = 0$ [see Eq. (4)], and if this resonant frequency coincides with the carrier frequency ω_0, then the expressions (3a) and (4) show that the zero-order current $\hat{I}_0(z,\omega)$ has two second order poles at $\omega = \pm\omega_0$. Physically this behavior of $\hat{I}_0(z,\omega)$ implies that both the time-domain current and the radiated field increase with time, like $t \sin\omega_0 t$, which cannot happen in practice. This non-physical result is contributed primarily by the unrealistic assumption of a voltage generator having a zero internal impedance. Furthermore, a higher-order approximation of the current $\hat{I}(z,\omega)$ contains in its denominator a term of the form $\cos\beta h + d_1(h, \omega)/\Omega + \ldots$, which does not vanish at $\omega = \pm\omega_0$. This situation is analogous to the transient response of a circuit consisting of a loss less inductance L and a capacitance C connected in series with a voltage generator (with zero internal impedance) producing a sinusoidal pulse $\sin \omega_0 t$ of duration T. If the circuit parameters, L and C are such that $\omega_0^2 = (1/LC)$, then the transient current will behave like $t \sin \omega_0 t$ and $(t - T) \sin \omega_0 t$. Since in reality, there is no inductance without resistance and no generator with zero internal impedance, such behavior of the current never occurs. Therefore, in order to be consistent with the zero-order approximation of the current distribution (4) and the assumption of a perfectly conducting thin cylindrical antenna, an inclusion of a non-zero internal impedance of the pulse generator in the present analysis is a meaningful and realistic way to remove the non-physical transient response. Although the internal impedance of a generator is a function of frequency, in general, we shall assume it to be independent of frequency. Let us, therefore, take the generator impedance $Z_g = \alpha Z_0$, where α is real, $(0 < \alpha < 1)$ and Z_0 is defined above. As a result the current distribution $\hat{I}_0(z,\omega)$ is modified as follows:

$$\hat{I}_{0,M}(z,\omega) = [\hat{V}(\omega)/Z_0] \cdot [\sin \beta(h - |z|)/\{\alpha \sin \beta h - i \cos \beta h\}], \tag{5a}$$

$$= [2i\hat{V}(\omega)/Z_0(1 + \alpha)] \cdot [\sin \beta(h - |z|)/\{1 + \Gamma \exp(-i2\beta h)\}] \cdot \exp(-i\beta h), \tag{5b}$$

$$= [2i\hat{V}(\omega)/Z_0(1 + \alpha)] \cdot [\sin \beta(h - |z|) \cdot \exp(-i\beta h)] \cdot \sum_{n=0}^{\infty} (-\Gamma)^n \exp(-i2n\beta h), \tag{5c}$$

where $\Gamma = (1 - \alpha)/(1 + \alpha) < 1$, is the reflection coefficient from the antenna to the generator. For a matched system $\alpha = 1$ and Γ becomes zero. The relation in (5) for the modified zero-order frequency response of the current was also used in Ref. [1], where the voltage source was a Dirac delta function in time.

The radiated electric field in the frequency, or Fourier-transform, domain due to a center-fed thin cylindrical antenna with current distribution given by (5), can be expressed in the following way.

$$\hat{E}_\theta(r, \theta, \omega) = [i\mu_0\omega \sin \theta/4\pi r] \cdot \exp(-i\beta r)\, g_{0,M}(\theta, \omega), \tag{6}$$

where

$$g_{0,M}(\theta, \omega) = \int_{-h}^{h} \hat{I}_{0,M}(z', \omega) \exp(i\beta z' \cos \theta) dz', \tag{7a}$$

$$= (4i\hat{V}(\omega)/[Z_0(1 + \alpha)]) \cdot [\{\cos(\beta h \cos \theta) - \cos \beta h\}/(\beta \sin^2 \theta)]$$

$$\cdot \sum_{n=0}^{\infty} (-\Gamma)^n \exp\{-i(2n + 1)\beta h\}. \tag{7b}$$

Then the time-dependent radiated electric field given by the inverse Fourier-transform of (6), can be expressed formally by

$$E_\theta(r, \theta, t) = (1/2\pi) \cdot \int_{-\infty}^{\infty} \hat{E}_\theta(r, \theta, \omega) \exp(i\omega t) d\omega. \tag{8}$$

Since we wish to express the time-dependent electric field in terms of unit step functions which can be used to display pulses conveniently, we shall use the Laplace transform and its inverse. Here the Laplace transform is obtained from the corresponding Fourier transform by replacing ω with $-is$, where s is the Laplace transform variable. As a result, the expression (8) can be replaced by

$$E_\theta(r,\ \theta,\ t) = (1/2\pi i) \int_C \hat{E}_\theta(r,\ \theta,\ -is) \exp(st) ds\,. \tag{9}$$

The contour C runs along the imaginary axis of the complex s-plane with indentation from the right-side at the poles $s = \pm i\omega_0$. Finally the contour is closed by a semi-circle in the left-half plane at infinity, so that the poles $s = \pm i\omega_0$ lie inside this closed contour. Then the time-dependent radiated electric field can be expressed in the following manner.

$$E_\theta(r,\theta,t) = [V_0\zeta_0/\{2Z_0(1+\alpha)\pi r \sin\theta\}]$$

$$\cdot \Bigg[\sum_{n=0}^{\infty} (-\Gamma)^n \sin\omega_0(t^* - 2nh/c) \cdot [U(t^* - 2nh/c) - U(t^* - 2nh/c - T)]$$

$$- \sum_{n=0}^{\infty} (-\Gamma)^n \sin\omega_0\{t^* - (2nh + h(1-\cos\theta))/c\}$$

$$\cdot [U\{t^* - (2nh + h(1-\cos\theta))/c\} - U\{t^* - (2nh + h(1-\cos\theta))/c - T\}]$$

$$- \sum_{n=0}^{\infty} (-\Gamma)^n \sin\omega_0\{t^* - (2nh + h(1+\cos\theta))/c\}$$

$$\cdot [U\{t^* - (2nh + h(1+\cos\theta))/c\} - U\{t^* - (2nh + h(1+\cos\theta))/c - T\}]$$

$$+ \sum_{n=0}^{\infty} (-\Gamma)^n \sin\omega_0\{t^* - 2(n+1)h/c\} \cdot [U\{t^* - 2(n+1)h/c\} - U\{t^* - 2(n+1)h/c - T\}] \Bigg]\,, \tag{10}$$

where $t^* = t - r/c$ = retarded time. Similarly, the time-dependent current distribution along the dipole corresponding to the frequency domain zero-order modified expression given by (5) can be represented as follows.

$$I_{0,M}(z,\ t) = (1/(2\pi i)) \cdot \int_C \hat{I}_{0,M}(z,\ -is)\, e^{st} ds\,, \tag{11}$$

$$= [V_0/Z_0(1+\alpha)] \cdot \Bigg[\sum_{n=0}^{\infty} (-\Gamma)^n \sin\omega_0\{t + (h - |z| - (2n+1)h)/c\}$$

$$\cdot [U\{t + (h - |z| - (2n+1)h)/c\} - U\{t + (h - |z| - (2n+1)h)/c - T\}]$$

$$- \sum_{n=0}^{\infty} (-\Gamma)^n \sin\omega_0\{t - (h - |z| + (2n+1)h)/c\}$$

$$\left. \cdot [U\{t - (h - |z| + (2n+1)h)/c\} - U\{t - (h - |z| + (2n+1)h)/c - T\}] \right] \qquad (12)$$

Although the exciting sinusoidal voltage has a finite duration T, both the time-dependent radiated electric field and the current along the dipole continue to exist for a longer period with diminished amplitudes due to multiple reflections (ringing) between the dipole antenna and the pulse generator. However, when the pulse generator is matched to the antenna, then $\alpha = 1$ and $\Gamma = 0$. Therefore, the corresponding time-dependent radiated field and the antenna current distribution expressions simplify themselves considerably as shown below.

$$E_\theta(r,\theta,t) = [V_0\zeta_0/(4Z_0\pi r\sin\theta)] \cdot [\sin\omega_0 t^* \cdot \{U(t^*) - U(t^* - T)\}$$

$$- \sin\omega_0\{t^* - h(1-\cos\theta)/c\} \cdot [U\{t^* - h(1-\cos\theta)/c\} - U\{t^* - h(1-\cos\theta)/c - T\}]$$

$$- \sin\omega_0\{t^* - h(1+\cos\theta)/c\} \cdot [U\{t^* - h(1+\cos\theta)/c\} - U\{t^* - h(1+\cos\theta)/c - T\}]$$

$$+ \sin\omega_0(t^* - 2h/c) \cdot \{U(t^* - 2h/c) - U(t^* - 2h/c - T)\}] \qquad (13)$$

$$I_{0,M}(z,t) = (V_0/2Z_0) \cdot [\sin\omega_0(t^* - |z|/c) \cdot \{U(t^* - |z|/c) - U(t^* - |z|/c - T)\}$$

$$- \sin\omega_0\{t^* - (2h - |z|)/c\} \cdot \{U(t^* - (2h - |z|)/c) - U(t^* - (2h - |z|)/c)\}]. \qquad (14)$$

The first term of the radiated field (13) resembles very much the sinusoidal voltage pulse, but retarded, and is radiated directly from the feed-point $z = 0$ of the dipole antenna. The second term of (13) represents radiation from the upper end $z = h$ of the dipole, whereas the third term can be identified as the radiated field from the lower end $z = -h$. The fourth or the last term of (13) is again radiated from the feed point $z = 0$, after the current undergoes reflection from $z = \pm h$. In otherwords, one half of the last term of (13) is contributed by the current which traveled from $z = 0$ to $z = h$ and then back to $z = 0$, and the other half is due to the current traveling from $z = 0$ to $z = -h$ and then back to $z = 0$. This behavior of the field indicates that radiation appears to emerge only from the discontinuities of the antenna.

The first term of (14) represents a current pulse at a given point $\pm z$ of the dipole antenna, for which the pulse has not yet reached the end points of the antenna. It may be noted here that time t in the expression for the current (14) is the real time or the retarded time at $r = 0$. The second term in (14) designates the current pulse at a point $\pm z$ on the antenna, after it has been reflected once from the end points $z = \pm h$.

Assuming that $T > 2h/c$, the relation (13) can be re-expressed using seven intervals, in some of which two or more pulses overlap. They can be written as follows.

$E_\theta(r, \theta, t)/[V_0\zeta_0/(4\pi Z_0 r)]$

$$= (1/\sin\theta)\cdot[\sin \omega_0 t^*], \quad 0 < t^* < h(1 - \cos\theta)/c$$

$$= (1/\sin\theta)\cdot[\sin \omega_0 t^* - \sin \omega_o\{t^* - h(1 - \cos\theta)/c\}], \quad h(1 - \cos\theta)/c < t^* < h(1 + \cos\theta)/c$$

$$= (1/\sin\theta)\cdot[\sin \omega_0 t^* - \sin \omega_0\{t^* - h(1 - \cos\theta)/c\}$$

$$- \sin \omega_0\{t^* - h(1 + \cos\theta)/c\}], \quad h(1 + \cos\theta)/c < t^* < 2h/c$$

$$= (1/\sin\theta)\cdot[\sin \omega_0 t^* - \sin \omega_0\{t^* - h(1 - \cos\theta)/c\} - \sin \omega_0\{t^* - h(1 + \cos\theta)/c\}$$

$$+ \sin \omega_0(t^* - 2h/c)], \quad 2h/c < t^* < T$$

$$= (1/\sin\theta)\cdot[-\sin \omega_0\{t^* - h(1 - \cos\theta)/c\} - \sin \omega_0\{t^* - h(1 + \cos\theta)/c\}$$

$$+ \sin \omega_0(t^* - 2h/c)], \quad T < t^* < T + h(1 - \cos\theta)/c$$

$$= (1/\sin\theta)\cdot[-\sin \omega_0\{t^* - h(1 + \cos\theta)/c\}$$

$$+ \sin \omega_0(t^* - 2h/c)], \quad T + h(1 - \cos\theta)/c < t^* < T + h(1 + \cos\theta)/c$$

$$= (1/\sin\theta)\cdot[\sin \omega_0(t^* - 2h/c)], \quad T + h(1 + \cos\theta)/c < t^* < T + 2h/c. \qquad (15)$$

Similarly, the current can be represented in the following three intervals.

$I_{0,M}(z, t)/[V_0/2Z_0]$

$$= \sin \omega_0(t - |z|/c), \quad |z|/c < t < (2h - |z|)/c$$

$$= \sin \omega_0(t - |z|/c) + \sin \omega_0(t + |z|/c), \quad (2h - |z|)/c < t < T + |z|/c$$

$$= \sin \omega_0(t + |z|/c), \quad T < |z|/c < t < T + (2h - |z|)/c. \qquad (16)$$

4. ACKNOWLEDGMENT

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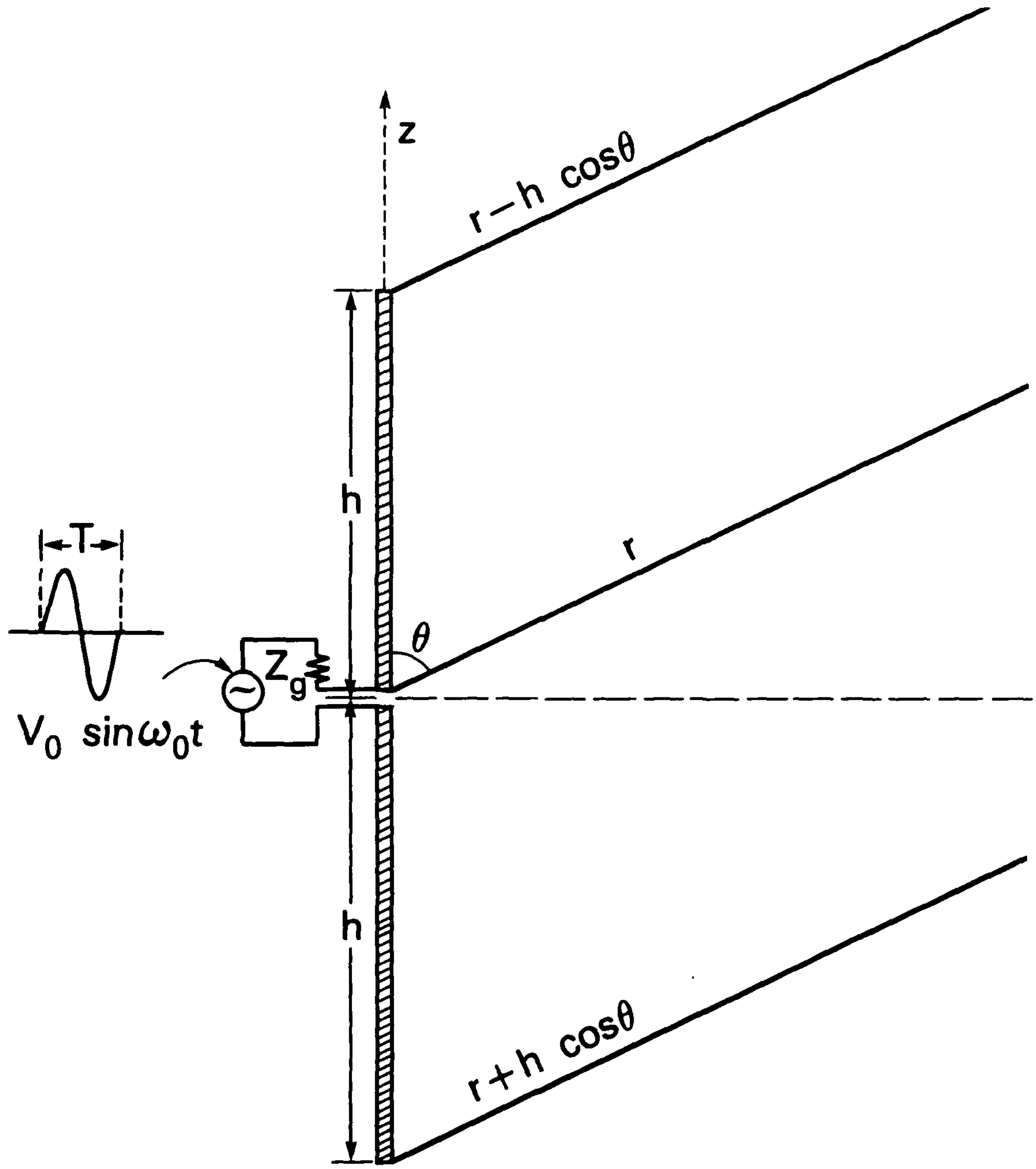

Fig. 1 — Transient Radiation from a Vertical Dipole Antenna Oriented Along the z-axis

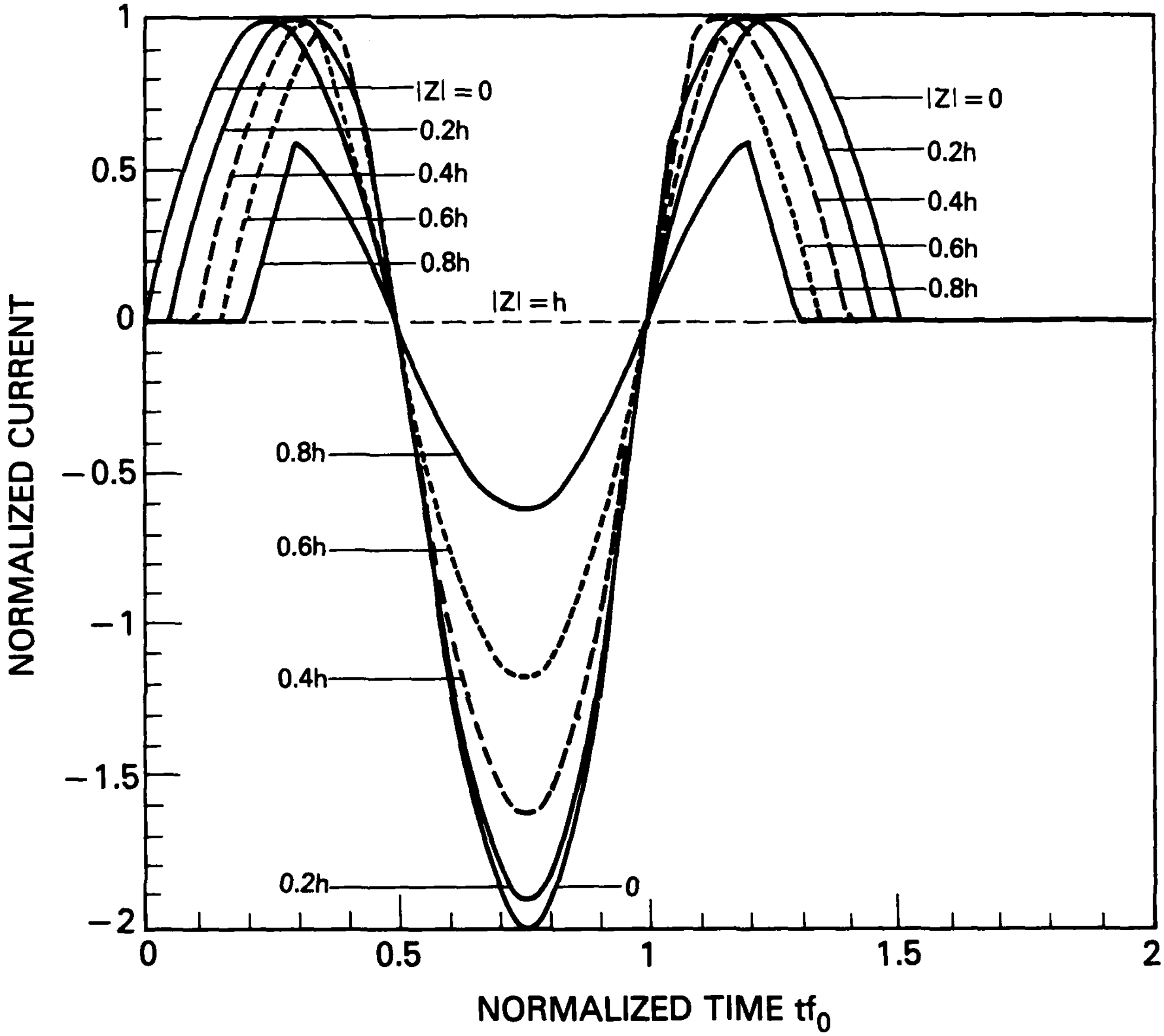

Fig. 2 — Normalized Transient Current Along a Half-Wave Dipole

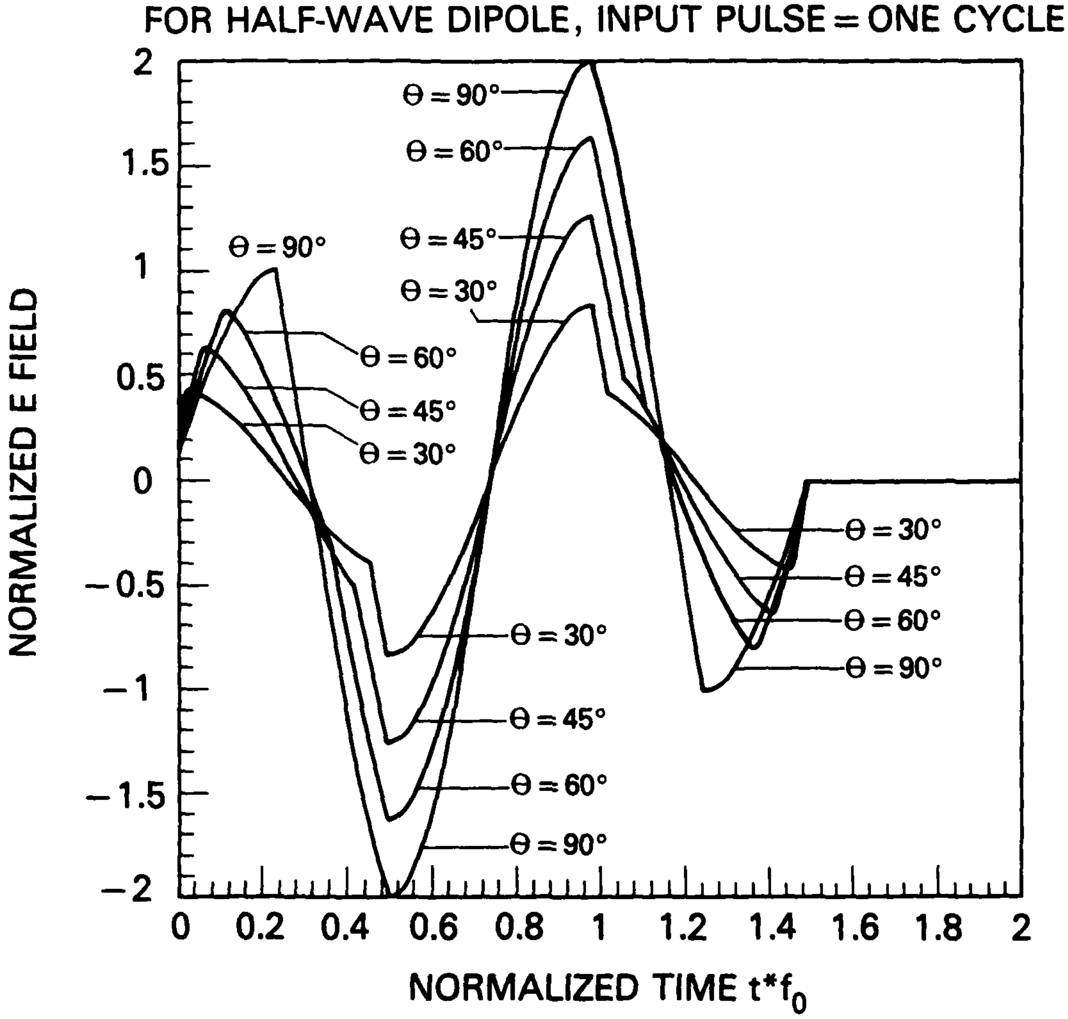

Fig. 3 — Transient Radiated Electric Field Observed at Different Angles from a Half-Wave Dipole

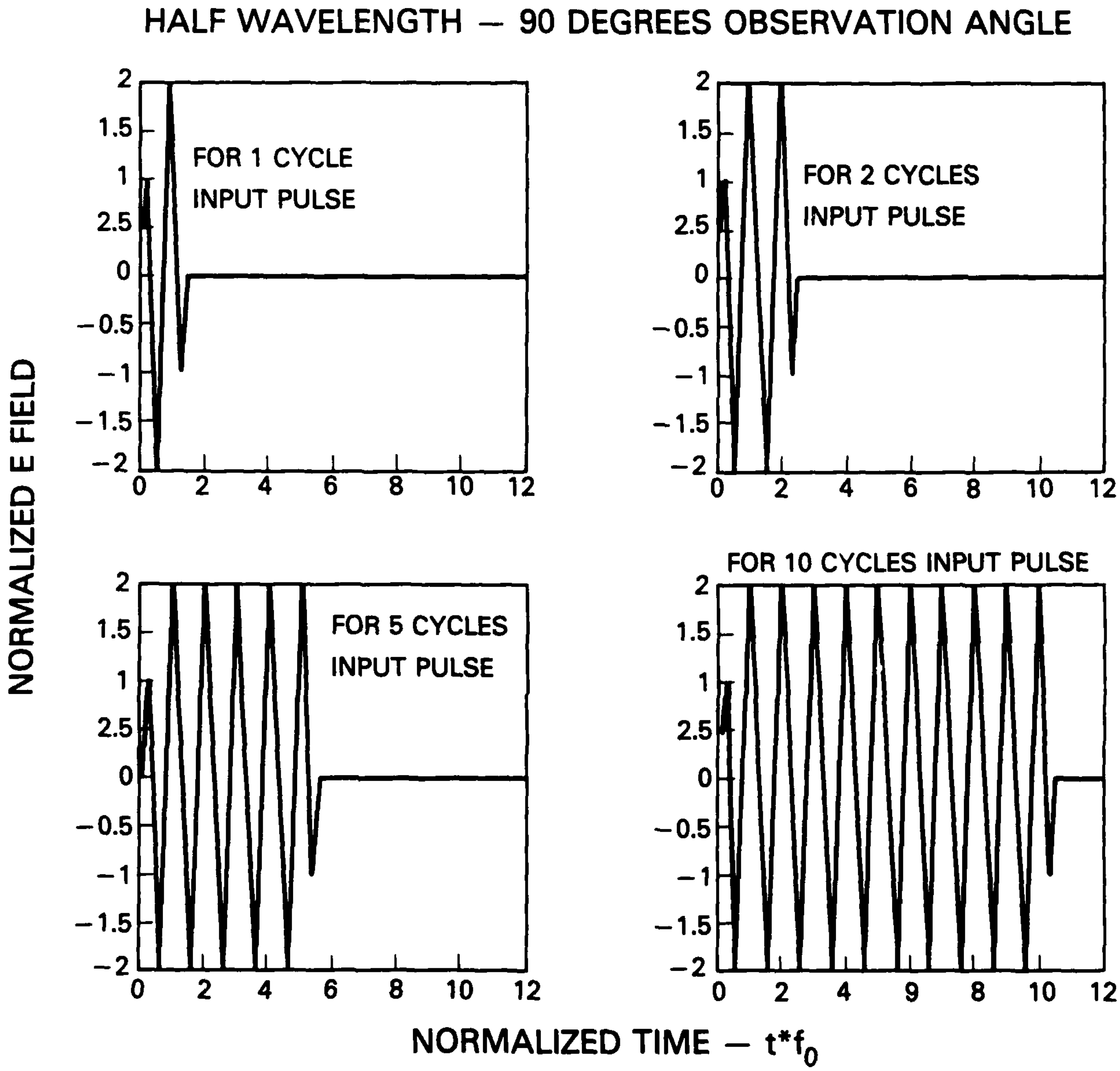

Fig. 4 — Transient Radiated Electric Field for Input Pulses with Different Widths. Half-Wave Dipole. Observation Angle = 90°

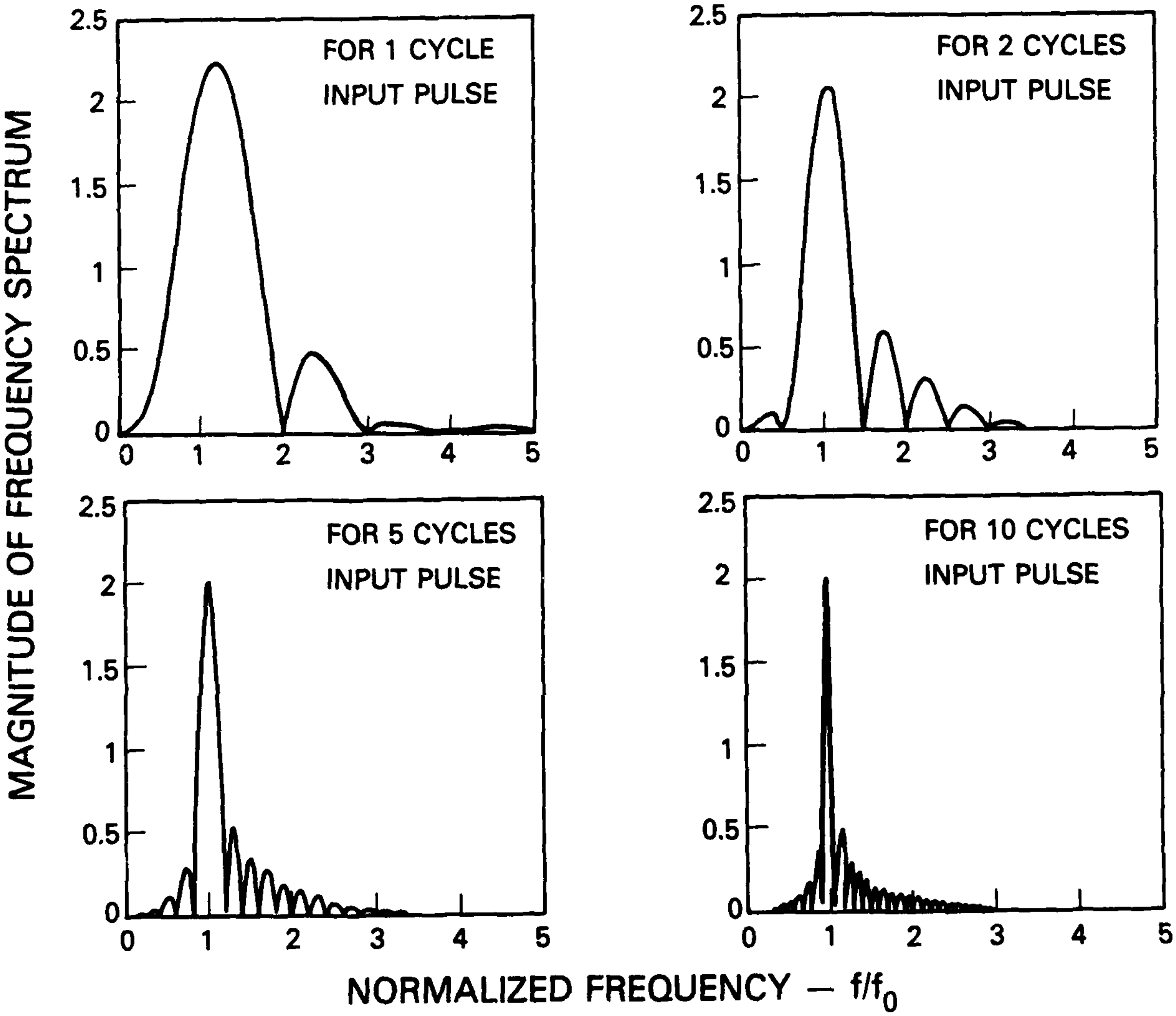

Fig. 5 — Frequency Response of Radiated Electric Field for Input Pulses with Different Widths. Half-Wave Dipole. Observation Angle = 90°

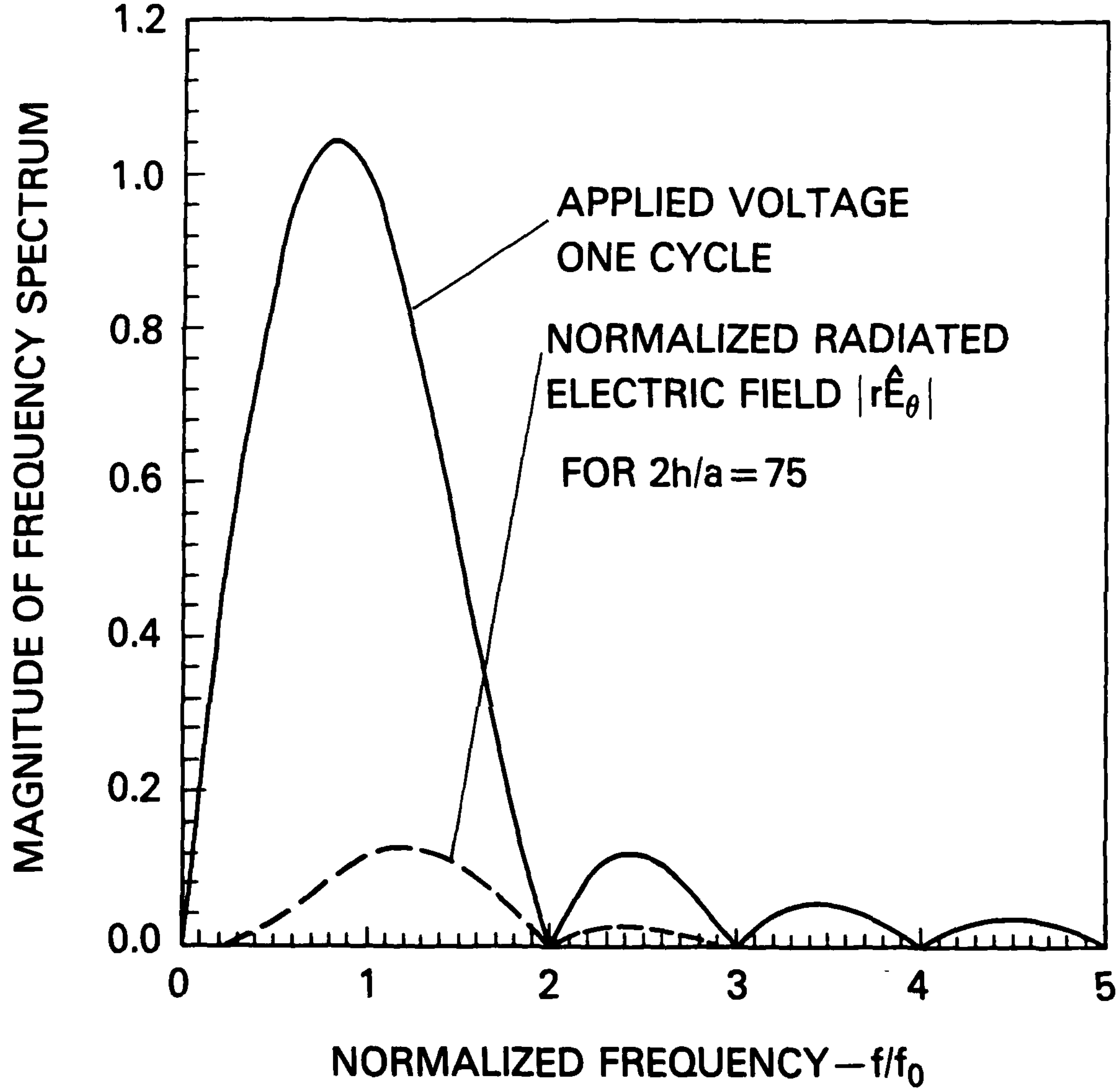

Fig. 6 — Spectrum of Applied Voltage (1 Cycle) and Radiated Electric Field for a Half-Wave Dipole. Observation Angle = 90°

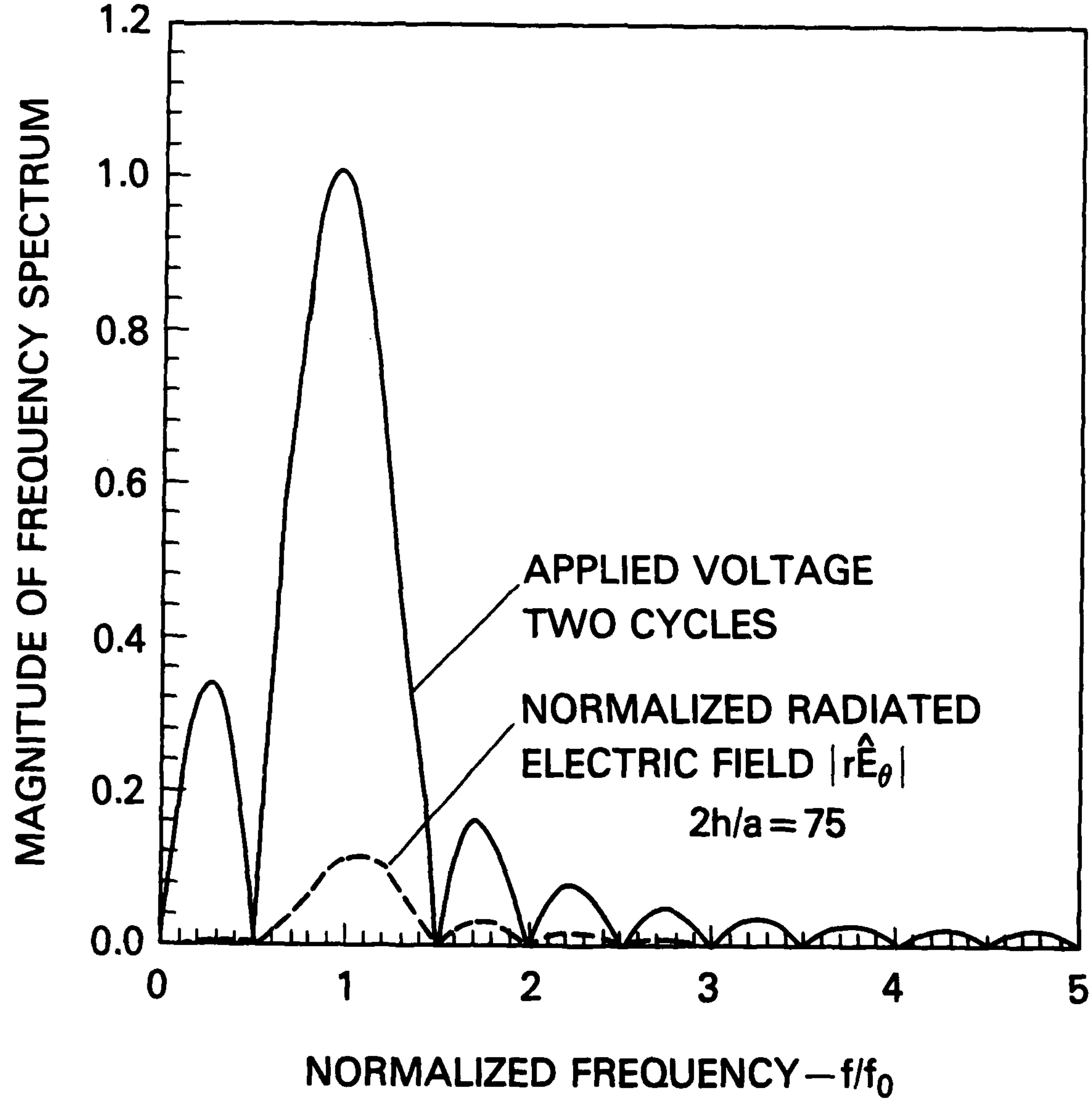

Fig. 7 — Spectrum of Applied Voltage (2 Cycles) and Radiated Electric Field for a Half-Wave Dipole. Observation Angle = 90°